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### INVESTIGATION OF AN ALTERNATIVE FINITE ELEMENT PROCEDURE: A ONE-STEP, STEADY-STATE ANALYSIS

An Investigation Conducted by:

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**ABSTRACT:** A one-step, steady-state finite element analysis procedure that is applicable to the problem of determining stresses and the driving force to pass a plane through soil at constant velocity is developed. This type of analysis is shown to be an alternative to doing a multi-step analysis using a commercial finite element code if it becomes computationally expensive. A steady-state prob-

lem allows the time dependence to be expressed as a space dependence and thus solved in one step. The feasibility of this type of analysis is demonstrated but more investigation is needed in soil failure at the plow tip and an associated soil plasticity model before any attempt is made to produce a production three-dimensional one-step code or to do a multi-step transient analysis using existing code.

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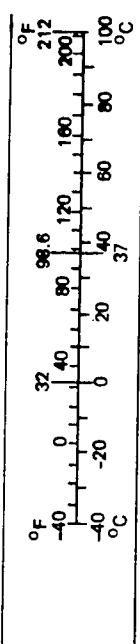
# METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures				Approximate Conversions from Metric Measures			
Symbol	When You Know	Multiply by	To Find	Symbol	When You Know	Multiply by	To Find
in ft yd mi	inches	2.5 30 0.9 1.6	centimeters	mm cm m km	millimeters	0.04 0.4 3.3 1.1 0.6	inches
	feet		centimeters		centimeters		inches
	yards		meters		meters		feet
	miles		kilometers		kilometers		yards miles
in <sup>2</sup> ft <sup>2</sup> yd <sup>2</sup> mi <sup>2</sup>	square inches	6.5 0.09 0.8 2.6 0.4	square centimeters	cm <sup>2</sup> m <sup>2</sup> km <sup>2</sup> ha	square centimeters	0.16 1.2 0.4 2.5	square inches
	square feet		square meters		square meters		square yards
	square yards		square meters		square kilometers		square miles
	square miles		square kilometers		hectares (10,000 m <sup>2</sup> )		acres
oz lb	ounces	28 0.45 0.9	grams	g kg t	grams	0.035 2.2 1.1	ounces
	pounds		kilograms		kilograms		pounds
	short tons (2,000 lb)		tonnes		tonnes (1,000 kg)		short tons
tsp Tbsp fl oz c pt qt gal ft <sup>3</sup> yd <sup>3</sup>	teaspoons	5 15 30 0.24 0.47 0.95 3.8 0.03 0.76	milliliters	ml l l l l l l m <sup>3</sup> m <sup>3</sup>	milliliters	0.03 2.1 1.06 0.26 35 1.3	fluid ounces
	tablespoons		milliliters		liters		pints
	fluid ounces		milliliters		liters		quarts
	cups		liters		liters		gallons
	pints		liters		cubic meters		cubic feet
	quarts		liters		cubic meters		cubic yards
	gallons		liters				
ft <sup>3</sup> yd <sup>3</sup>	cubic feet	0.03 0.76	cubic meters	m <sup>3</sup> m <sup>3</sup>			
	cubic yards		cubic meters				
TEMPERATURE (exact)				TEMPERATURE (exact)			
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature

6	23	Symbol	When You Know	Multiply by	To Find	Symbol
8	21		mm	LENGTH	inches	in
	20		cm	0.04	inches	in
7	19		m	0.4	feet	ft
	18		m	3.3	yards	yd
17	17		km	1.1	miles	mi
	16			0.6		
16	16			AREA		
15	15		cm <sup>2</sup>	0.16	square inches	in <sup>2</sup>
14	14		m <sup>2</sup>	1.2	square yards	yd <sup>2</sup>
	13		km <sup>2</sup>	0.4	square miles	mi <sup>2</sup>
13	13		ha	2.5	acres	
5	12			MASS (weight)		
	11		g	0.035	ounces	oz
	10		kg	2.2	pounds	lb
4	9		t	1.1	short tons	
	8			VOLUME		
3	7		ml	0.03	fluid ounces	fl oz
	6		l	2.1	pints	pt
	5		l	1.06	quarts	qt
	4		l	0.26	gallons	gal
	3		m <sup>3</sup>	35	cubic feet	ft <sup>3</sup>
	2		m <sup>3</sup>	1.3	cubic yards	yd <sup>3</sup>
	1			TEMPERATURE (exact)		
			°C	9/5 (then add 32)	Fahrenheit temperature	°F

1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price \$2.25, SD Catalog No. C13.10:286.

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## Introduction

The purpose of this study was to investigate the feasibility of developing a one-step, steady-state finite element analysis procedure that is applicable to the "plow problem". The plow problem being defined as the determination of the state of stress and deformation induced in a soil deposit by the passage of a plow type device through the soil at a constant velocity and the calculation of the force required to drive the plow. The analysis of this problem can be carried out as a transient analysis using available commercial finite element codes. Such an analysis would, in a step wise fashion, analyze the configuration as the plow entered the soil and proceeded to reach a steady-state plowing action. The advantage of such an analysis is that it can be performed using available software. In addition, if the plow insertion is described in a realistic fashion, the stresses and deformations developed during the insertion process would also be determined. The only possible drawback to such an analysis is possible large computational costs associated with a transient, nonlinear, inelastic, three-dimensional finite element analysis of a relatively complicated configuration.

Assuming that the steady-state plowing process does not result in any periodic localization (e.g., cracks propagating into the soil mass at regular intervals from the plow path), an alternative would be to perform a one-step, steady-state analysis. An observer situated on a plow passing through a homogeneous soil mass at a constant velocity would observe steady-state conditions. The goal of a steady-state analysis is to capture this steady-state behavior in a one step analysis. The advantage of such an analysis is that multiple solution steps are not required and, thus, there is a potential for considerable computational cost saving. The disadvantage is that when the problem is nonlinear due either to material or geometric nonlinearities, commercial software does not exist to perform the analysis. Approximate analyses [1,2] for specialized geometric configurations have been reported for a related class of Geotechnical Engineering problems (insertion of a sampling tube into a soil mass), however, in order to determine the deformation pattern the method treats the soil as a liquid; the degree of approximation

introduced by this assumption is hard to access. In, addition, no general procedures are available to extended this type of analysis to the plow problem.

Weighting the advantages and disadvantages of the two competing analysis procedures it seemed clear that the available and proven multi-step, transient analysis procedure is the preferable way to proceed for a project requiring a solution within a given time frame. However, as a possible backup in case the transient analysis should prove to be excessively expensive, it seemed advisable to perform a preliminary investigation of the feasibility of the steady-state procedure. Such a preliminary investigation was the goal of this portion of the project.

### Scope of Study

In order to model the plowing process as steady state the following assumptions (these may be viewed as restrictions or approximations) are required:

- 1) It is assumed that the soil deposit is infinite in extent, level and homogeneous in the direction of the plowing action.
- 2) It is assumed that the plow moves at a constant depth, with constant velocity  $v_0$  and in a straight path.
- 3) It is assumed that no significant "periodic localizations" occur in the soil mass during the plowing process. (If significant "periodic localizations", should occur, then an observer on the plow would not observe steady-state conditions.) An example of a periodic localization would be the radiation, from the plow path, of cracks into the surrounding soil at regular intervals along the path.

The objective of this study was to demonstrate the feasibility of the one-step, steady-state analysis procedure for the plow problem. In order to carry out the investigation within the time and financial constraints of the project, additional restrictions were introduced in order to simplify the analysis. However, it should be emphasized that these restrictions are not required for a steady-state analysis to apply. It is hoped that even within the confines of these simplifying assumptions that all the basic features of a one-step, steady-state analysis can be demonstrated and that success for this restricted class of problems will indicate the likelihood of success if the method were to be applied to the actual plow problem.

The additional simplifying assumptions are:

- 1) The geometry will be modeled as 2-dimensional (plane stress or plane strain) rather than 3-dimensional as is the case for the actual plow problem.
- 2) It will be assumed that the shape of the plow is such that the soil remains in contact with the plow along its entire surface and that no crack in the soil propagates and opens ahead of the plow.
- 3) It is assumed that the velocity  $v_0$  is sufficiently large and the permeability of the soil is sufficiently small, that the soil mass can be assumed to be "undrained".
- 4) The soil will be modeled as a linear viscoelastic material

The two neglected phenomenon of pore water flow due to the development of excess pore water pressure and the elastic-plastic behavior of real soil, both lead to a history dependency of the solution. The assumed linear viscoelasticity of the soil behavior also leads to a history dependency of the solution. It is not intended to suggest that these two history dependencies are in any way equivalent (although one might try to calibrate the viscoelasticity model so as to crudely capture the effects of redistribution of pore water pressure and soil plasticity). Instead it is hoped that the demonstration of the ability of the steady-state analysis to capture the history dependency of the viscoelastic properties will demonstrate the feasibility of capturing the history dependency of pore pressure redistribution and soil plasticity by means of a steady-state analysis.

While it is anticipated that inertia effects will only be marginally important for the plow speeds of interest in this study, they are included in order to demonstrate the ease with which they may be modeled.

### Definition of Problem

The purpose of this steady-state analysis is to study the disturbance in a soil mass produced by a plow and to determine the force required to drive the plow. A simple two-dimensional configuration of such a process is shown in Figure 1. The design parameters are the shape of the plow  $g$ , the coefficient of friction,  $f$ , between the plow and the soil, and the velocity of the plow  $v_0$ .

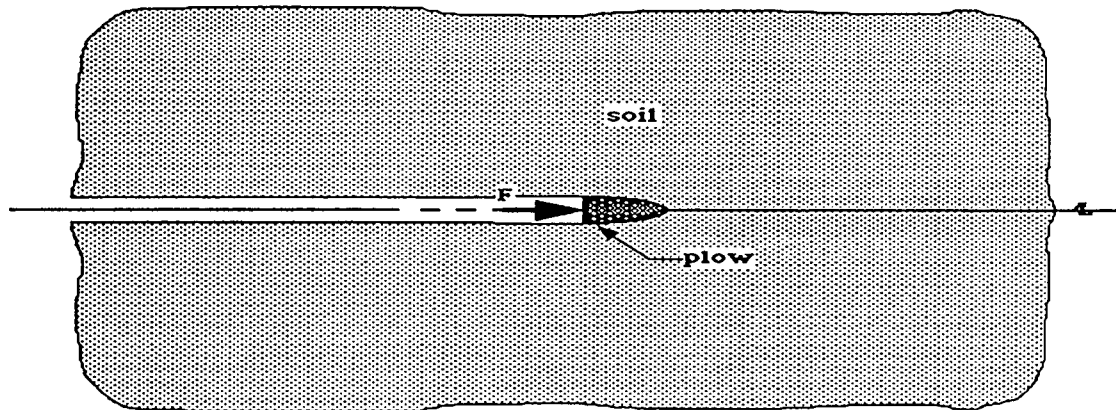


Figure 1. Two-dimensional plowing of a soil mass

### Steady-State Behavior

The initial insertion of the plow into the soil is a transient process which is not addressed in this study, i.e., it is assumed that the plowing process has proceeded for a long enough period of time that steady-state conditions have been reached. The analysis looks at the process at a particular instant in time,  $t=T$ , when the plow has reached the point, relative to the fixed x-y coordinate system, shown in Figure 2 (an alternative point of view is to consider the

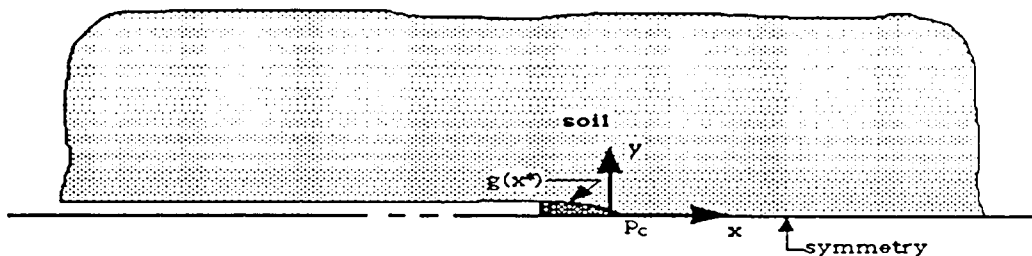


Figure 2. Plowing process at Time T

coordinate system as fixed to the moving plow). Point  $p_c$  denotes the point in the soil that is at the tip of the plow at the instant of time of interest  $T$ . Its location in space in the undeformed soil has been selected to be the origin of the  $x$ - $y$  coordinates. The displacement the soil point in question experiences in going from its undeformed location to its deformed location is  $u_c$ , see Figure 3.

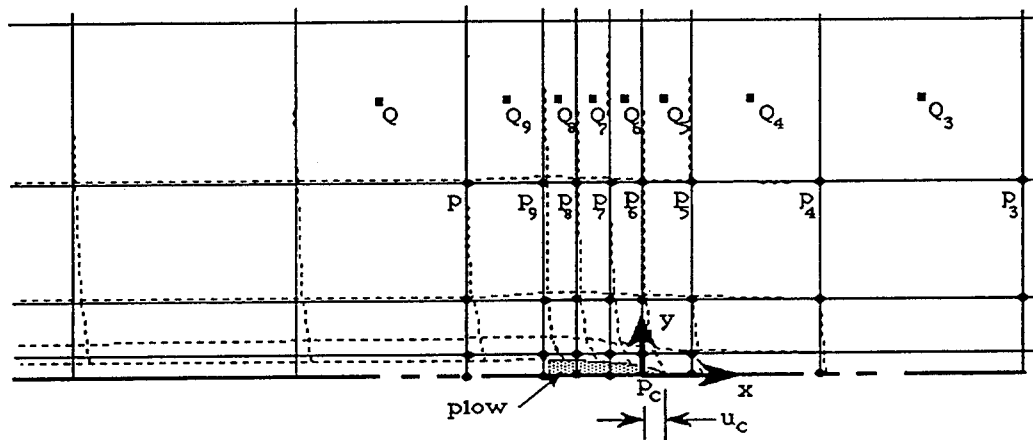


Figure 3. Deformed and undeformed grids at Time  $T$

In Figure 3 is shown a simple rectangular grid attached to the soil. Two views are shown, one before the soil is disturbed by the plow (undeformed grid) and the second of the deformed soil at the instant of time of interest,  $T$ . Consider the generic node point  $P$  fixed to the soil. With time, as the plow moves through the soil, the locations of points  $P_9, P_8, P_7, \dots$  ( $P_1$  being the last node point in the sequence at the right extreme of the grid) will successively bear the same relationship to the location of the plow as  $P$  does currently. Thus, if one wishes to know the past history experienced by the soil at  $P$  one merely needs to look at the present states of points  $P_n$ . A similar statement may be made for the sequence  $Q, Q_9, Q_8$ , etc.

Specifically the state of the soil at point  $P$  (located at  $x, y$ ) at a past time  $T - \Delta t$  is identical to the current state of the soil at a point located at  $x + v_0 \Delta t, y$ . Thus, for example the strain  $\{\epsilon\}$  of the soil at location  $x, y$  and at time  $T - \Delta t$  is equal to

the strain at point  $x+v_0\Delta t, y$  at time  $T$  (where  $\{\epsilon\}$  contains the strain components,  $\epsilon_{ij}$ , written in vector form) , i.e.,

$$\{\epsilon(x,y,T-\Delta t)\} = \{\epsilon(x+v_0\Delta t,y,T)\} \quad (1)$$

Thus, partial derivatives with respect to time are equal to the partial derivatives with respect to  $x$ , e.g.,

$$\frac{\partial u}{\partial t} = -v_0 \frac{\partial u}{\partial x} \quad (2)$$

### Governing Equations

The study will be limited to small rotation and small deformation conditions. The equations of motion are:

$$\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = \rho \frac{\partial^2 u_x}{\partial t^2} \quad (3)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} = \rho \frac{\partial^2 u_y}{\partial t^2}$$

Using the results of eq (2) permits the accelerations to be replaced by space derivatives, yielding:

$$\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} - \rho v_0^2 \frac{\partial^2 u_x}{\partial x^2} = 0 \quad (4)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} - \rho v_0^2 \frac{\partial^2 u_y}{\partial x^2} = 0$$

The deformation is described by the strain terms:

$$\begin{aligned} \epsilon_{xx} &= \frac{\partial u_x}{\partial x} \\ \epsilon_{yy} &= \frac{\partial u_y}{\partial y} \\ \tau_{xy} &= \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \end{aligned} \quad (5)$$

For an inelastic material, the state of stress at a particular point  $x,y$ , and at the current time  $T$ , is a function of the history of deformation at the point in question, i.e.,

$$\{\sigma(x,y,T)\} = \{\sigma(\{\epsilon(x,y,t=-\infty \rightarrow T)\})\} \quad (6)$$

Where  $\{\sigma\}$  is the stress written in vector form. The actual form of this dependency depends upon the form of the material model used for the soil. In this study a linear viscoelastic law is used (e.g., see [3]).

Using eq (1) the time dependence in eq (6) can be expressed as a space dependence

$$\{\sigma(x,y,T)\} = \{\sigma(\{\epsilon(x'=x \rightarrow \infty, y,T)\})\} \quad (7)$$

The actual range of  $x'$  must only be carried far enough that undisturbed (by the action of the plow) soil is reached.

### Cutting Condition

The soil is sliced to form two new surfaces at the tip of the plow. Thus, in the soil right at the tip of the plow some critical state for new surface formation must exist, this will be referred to as the "cutting criterion" and must be specified in the finite element analysis of the plowing process. There is a problem, however, in that very little appears to be known about this criterion. Considerable work has been done concerning the nature of the failure state at the tip of an advancing free surface crack in materials such as metals, however, very little appears to be known for the corresponding state in soil where the advancing crack tip is being forced by a cutting tool. In the absence of any concrete information, two possible criteria will be considered. The first is that the normal effective stress across the tip of the advancing cut is zero. As an alternative, the second criterion is that the soil must be in a state of failure, as defined by the critical state line, at the tip of the advancing cut. The equation forms of these two criteria will be discussed later.

### Finite Element Analysis

The analysis will typically be nonlinear due to a) the nonlinear nature of the constitutive equations that are used to model soil behavior (for the purpose of this study a linear law is used, however, in general soil behavior is nonlinear), b) the cutting criterion for the soil at the tip of the plow, and c) the frictional behavior occurring at the soil-plow interface. A modified Newton-Raphson solution scheme will be used to handle the nonlinearities. It is

called a "modified" scheme because approximations are introduced into the Jacobian. These approximations have no effect on accuracy (as long as convergence is achieved) but do slow down the rate of convergence. The desirability of introducing these approximations is explained as the analysis is developed.

The iterative estimate for the vector of node point degrees of freedom (displacements in the conventional displacement approach adopted here for the undrained problem) at iteration I is written as  $\{u\}^{(I)}$ . The Newton-Raphson correction to the I-1 estimate that gives the I<sup>th</sup> estimate is (the initial estimate  $\{u\}^{(0)}$  is taken to be zero).

$$\{u\}^{(I)} = \{u\}^{(I-1)} - [J]^{(I-1)^{-1}} \{R\}^{(I-1)} \quad (8)$$

Where  $\{R\}^{(I-1)}$  is the residual vector for the finite element equations as given by the (I-1) solution. The approximate Jacobian is denoted by the matrix  $[J]$ . The elements of the Jacobian are the first derivatives of the residuals with respect to the node point displacements; they are evaluated using the (I-1) estimate for the solution.

Whether or not convergence will occur and if so the speed of convergence depends in part upon the accuracy of the approximate Jacobian  $[J]$ . However, if convergence does occur, obtaining the correct solution (leaving aside any questions of non-uniqueness) only depends upon using the correct expression for the residual vector.

The finite element equations are obtained using Galerkin's weighted residual method applied to the negatives of the equations of motion, eq(4) (the negative sign is used to preserve the sign convention used in a minimum potential energy formulation). The two weighted residuals associated with node n are (the same base functions  $\Phi_n(x,y)$  are used to approximate both displacement components  $u_x$  and  $u_y$ ; the base function  $\Phi_n$ , for node n, is made up of all the element shape functions connected to node point n; the subscripts 2n-1 and 2n give the locations of the two rows in the global residual vector):

$$R_{2n-1} = \iint - \left[ \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} - \rho v_0^2 \frac{\partial^2 u_x}{\partial x^2} \right] \Phi_n dx dy$$

$$R_{2n} = \iint - \left[ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} - \rho v_0^2 \frac{\partial^2 u_y}{\partial x^2} \right] \Phi_n dx dy \quad (9)$$

In the above equations, the terms containing derivatives of the stresses are integrated by parts. This integration by parts produces element interface terms to cancel the residuals in the interface stress equilibrium condition (continuity of the traction vector across element boundaries) which are implicitly present in the above residual expressions, the results are:

$$R_{2n-1} = \iint - \left[ \tau_{xx} \frac{\partial \Phi_n}{\partial x} + \tau_{xy} \frac{\partial \Phi_n}{\partial y} + \rho v_o^2 \frac{\partial^2 u_x}{\partial x^2} \Phi_n \right] dx dy$$

$$R_{2n} = \iint - \left[ \tau_{xy} \frac{\partial \Phi_n}{\partial x} + \tau_{yy} \frac{\partial \Phi_n}{\partial y} + \rho v_o^2 \frac{\partial^2 u_y}{\partial x^2} \Phi_n \right] dx dy$$
(10)

In a conventional finite element analysis, the integrations in the above expressions are carried out element by element and give rise to the element contributions to the global residual vector (these element contributions are often called the element load vectors). Approximate (reduced) integration is introduced into this process. For a four node, bi-linear, isoparametric element the stress terms are integrated using one point integration at the element center (i.e., at the origin of the isoparametric coordinates for the element). This one point integration is used to avoid element "lock-up" due to the nearly incompressible behavior of undrained soil problems. Because this reduced integration may introduce hour-glassing into the solution, hour-glass control will be used (e.g., see [4]). The inertia effects are integrated using quadrature points only at the nodes; this process leads to a lumped mass idealization which has proven advantageous in many problems in dynamics.

This partitioning of the integration into two parts, leads to the assembly of the finite element equations by separately considering element contributions and node contributions. The contribution of an element will be contained in the element matrices (the element residual and Jacobian matrices, alternatively denoted as the element load and stiffness matrices).

From the stress terms of the above equations the components of the element residual vector,  $R_e$ , are found (where the index  $j$  runs from 1 to 4,  $c$  denotes the element center,  $N_j$  are the bi-linear shape functions for the isoparametric element,  $A$  is the area of the element, and " $\}_c$ " denotes evaluation at the element center):

$$R_{e2j-1} = A \left\{ \tau_{xx} \frac{\partial N_j}{\partial x} + \tau_{xy} \frac{\partial N_j}{\partial y} \right\}_c$$

$$R_{e2j} = A \left\{ \tau_{xy} \frac{\partial N_j}{\partial x} + \tau_{yy} \frac{\partial N_j}{\partial y} \right\}_c$$
(11)

It is convenient to express the above equations in matrix form. Denote the derivatives of the shape function  $N_i$  with respect to  $x$  and  $y$  as  $F_i$  and  $G_i$  respectively. The vector of element residuals ( $\langle Re \rangle = \langle Re_1, Re_2, Re_3, \dots, Re_8 \rangle$ ) can then be written as:

$$\{Re\} = A [B]_c^T \{\sigma\}_c \quad (12)$$

Where  $\{\sigma\}$  is the stress components  $\tau_{ij}$  written in vector form and the matrix  $[B]$  is:

$$[B] = \begin{bmatrix} F_1 & 0 & F_2 & 0 & F_3 & 0 & F_4 & 0 \\ 0 & G_1 & 0 & G_2 & 0 & G_3 & 0 & G_4 \\ G_1 & F_1 & G_2 & F_2 & G_3 & F_3 & G_4 & F_4 \end{bmatrix} \quad (13)$$

The strain vector  $\{\epsilon\}$  ( $\langle \epsilon \rangle = \langle \epsilon_{xx}, \epsilon_{yy}, \gamma_{xy} \rangle$ ) can be expressed in terms of the  $[B]$  matrix and the vector of node point displacements ( $\langle U \rangle = \langle u_{x1}, u_{y1}, u_{x2}, \dots, u_{y4} \rangle$ ):

$$\{\epsilon\} = [B] \{U\} \quad (14)$$

For infinitesimal strains, the increment in strain resulting from a displacement increment is given by an equation similar to eq (14).

The element Jacobian (stiffness matrix for a linear problem) is found by differentiating the residual vector, eq (12), with respect to the node point displacements. Before this can be done the stresses must be explicitly expressed as a function of the strains. This process, for a steady-state problem, presents some difficulty. The problem is that the history dependence on the strain, for a steady state problem, has been converted to a space dependence, see eq(7). The result is that the stress at a point such as  $Q$  in Figure 3 is a function of the strains at points  $Q_1, Q_2, Q_3, \dots, Q$ . Because of this dependency, the residual for element  $Q$  is a function of not only the displacements of the nodes connected to  $Q$ , but also of all the displacements of the nodes connected to the elements  $Q_i$  to the right of  $Q$ . If the nodes are

numbered in the usual manner for a conventional finite element analysis so as to minimize the bandwidth of the equations, this dependency may result in the exact Jacobian not being banded. In order to avoid this possibility, an approximate Jacobian is used. (Further study may yield a more effective way of handling this problem.) The approximate Jacobian is developed by writing an approximation to eq (7) in the form:

$$\{\sigma(Q)\} \sim \{\sigma(Q)\}^{(I-1)} + [D(Q)]^{(I-1)} \{\Delta\epsilon(Q)\} \quad (15)$$

In the above equation the stress at the center of a given element (such as point Q in Figure 3) is approximated in terms of the value predicted using the strains from the previous iteration (I-1), for points Q<sub>1</sub>, Q<sub>2</sub>, Q<sub>3</sub>.....Q, plus a change induced by a change (during iteration I) in strain at point Q. The incremental change in stress resulting from the incremental change in strain at point Q is predicted using the tangent stiffness matrix [D]<sup>(I-1)</sup> (i.e.,  $\{\Delta\sigma\} = [D]\{\Delta\epsilon\}$ ). The tangent stiffness matrix is given by the algorithm that evaluates the material model used to represent the inelastic behavior of the soil (note that for general inelasticity the matrix [D] will not be symmetric). The expression in eq (15) is an approximation as it neglects the change in stress at point Q induced by the changes in the strains at points Q<sub>1</sub>, Q<sub>2</sub>, Q<sub>3</sub>..... As noted previously as long as convergence is achieved the use of an approximation Jacobian will have no affect on the accuracy of the converged solution. The net effect of this step is to approximate the space dependence in eq (7) by "successive substitution" and not by a true Newton-Raphson procedure (for some of the other nonlinear aspects of the problem the correct contributions to the Jacobian are used, so the overall analysis will be a mixture of successive substitution and true Newton-Raphson).

Introducing eqs (14) and (15) into eq (12) and differentiating with respect to the node point displacements leads to the usual tangent stiffness matrix (approximation to the true Jacobian):

$$[J_e] = A [B]_c^T [D]_c [B]_c \quad (16)$$

The above discussion concerning the use of the sequence of points Q<sub>1</sub>, Q<sub>2</sub>, Q<sub>3</sub>.....Q to evaluate history dependency, suggests that the use of a mesh where the grid lines in the x direction are straight and parallel to the x axis is advantageous (see Figure 4 for an example of a very course grid). With this property the points Q<sub>i</sub> can all be located at element centers (if such a mesh is not used an interpolation scheme must be used to find the strains at points Q<sub>1</sub>, Q<sub>2</sub>, Q<sub>3</sub>... , which all lie at the same distance y from the axis of symmetry as does point Q, in terms of the strains at the centers of the surrounding elements). For simplicity the remainder of the analysis is restricted to meshes of the type shown in Figure 4.

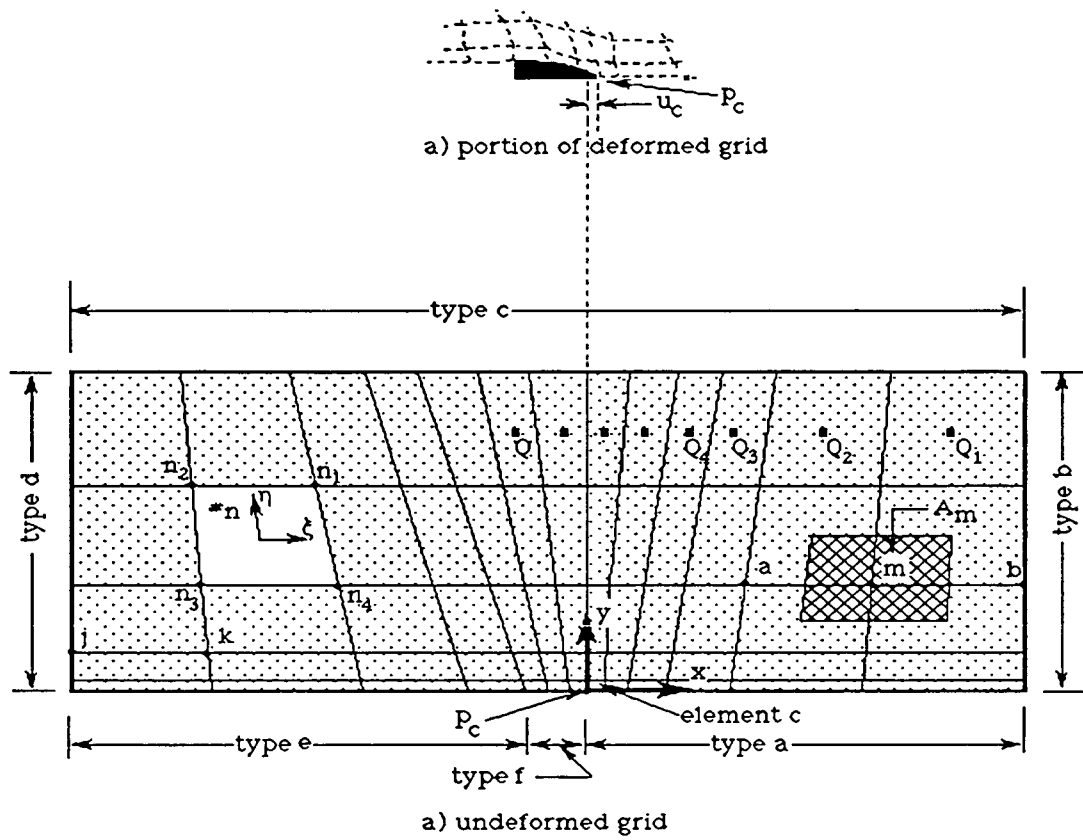


Figure 4. Deformed and undeformed grids for 2-D flow problem

Considering element "n" of Figure 4, denote the  $x, y$  coordinates of the connecting nodes as  $x_i, y_i$  (with node  $n_1$  selected to be the upper right one) ; the isoparametric coordinates are  $\xi_i=(1,-1,-1,1)$  and  $\eta_i=(1,1,-1,-1)$ . The well known isoparametric transformation can be easily specialized for the special element type of Figure 4 and shown to yield:

$$N_{ic} = 1/4$$

$$F_{ic} = \xi_i / \alpha_2 \quad (17)$$

$$G_{ic} = (\eta_i - \xi_i \alpha_1 / \alpha_2) / 2 \Delta y$$

Where (the element area is A):

$$\begin{aligned}
\alpha_1 &= x_1 + x_2 - x_3 - x_4 \\
\alpha_2 &= x_1 - x_2 - x_3 + x_4 \\
\alpha_3 &= x_1 - x_2 + x_3 - x_4 \\
\Delta y &= y_1 - y_4 \\
A &= \Delta y \alpha_2 / 2
\end{aligned} \tag{18}$$

For later use the values of  $F_i$ ,  $G_i$  and  $N_i$  at the corner  $k$  of the element are needed ( $\delta_{ik}$  is the Kronecker delta, i.e., the identity matrix):

$$\begin{aligned}
N_{ik} &= \delta_{ik} \\
F_{ik} &= \xi_i (1 + \eta_i \eta_k) / (\alpha_2 + \eta_k \alpha_3) \\
G_{ik} &= [\eta_i (1 + \xi_i \xi_k) - \xi_i (1 + \eta_i \eta_k) (\alpha_1 + \eta_k \alpha_3) / (\alpha_2 + \eta_k \alpha_3)] / 2 \Delta y
\end{aligned} \tag{19}$$

The element Jacobian and residual matrices (eqs 12 and 16) are assembled in the usual way using the "direct stiffness" concept; to this must be added the contributions from the integration of the inertia terms in eq (10) using quadrature points placed at the nodes.

Consider the node "m" in Figure 4 surrounded by the area  $A_m$  with nodes "a" and "b" directly to the left and to the right. A central finite difference operator is used to approximate the second derivatives with respect to  $x$  at point m:

$$\begin{aligned}
\frac{\partial^2 u_x}{\partial x^2} &\approx c_{-1} u_{x_a} + c_0 u_{x_m} + c_1 u_{x_b} \\
\frac{\partial^2 u_y}{\partial y^2} &\approx c_{-1} u_{y_a} + c_0 u_{y_m} + c_1 u_{y_b}
\end{aligned} \tag{20}$$

where

$$\begin{aligned}
c_{-1} &= \frac{2}{(x_m - x_a)(x_b - x_a)} \\
c_0 &= \frac{-2}{(x_m - x_a)(x_b - x_m)} \\
c_1 &= \frac{2}{(x_b - x_m)(x_b - x_a)}
\end{aligned} \tag{21}$$

The residual contributions of the last terms in eq (10) at node m, can now be evaluated ( $u_x$  and  $u_y$  are the I-1 estimates):

$$R_{\text{node}2m-1} = \omega (c_{-1} u_{x_a} + c_0 u_{x_m} + c_1 u_{x_b}) \quad (22)$$

$$R_{\text{node}2m} = \omega (c_{-1} u_{y_a} + c_0 u_{y_m} + c_1 u_{y_b})$$

where

$$\omega = A_m \rho v_0^2$$

The contributions to the Jacobian ( $J_{\text{node}i,j}$  denotes the contribution to the  $i,j$  term in the matrix  $[J]$ ) are found by differentiating the residual contributions with respect to the displacement components:

$$\begin{aligned} J_{\text{node}2m-1, 2a-1} &= \omega c_{-1} \\ J_{\text{node}2m-1, 2m-1} &= \omega c_0 \\ J_{\text{node}2m-1, 2b-1} &= \omega c_1 \\ J_{\text{node}2m, 2a} &= \omega c_{-1} \\ J_{\text{node}2m, 2m} &= \omega c_0 \\ J_{\text{node}2m, 2b} &= \omega c_1 \end{aligned} \quad (23)$$

Special consideration is required when "m" is a node on either the left or right boundary of the mesh (i.e., either no node point "a" to the left or "b" to the right exist). It is required that the horizontal dimensions of the soil mass being analyzed be selected large enough that at the very left the soil has come to rest and, thus, there is no acceleration while at the right the soil has as yet not felt the presence of the advancing plow and, thus, no acceleration exists. Hence, the node contributions to the residual and Jacobian matrices are zero at these points.

After the contributions of all elements and node points to the global Jacobian and residual matrices have been assembled, the resulting matrices must be modified to account for boundary conditions before the solution of eq (8) is undertaken. In Figure 4 the several different boundary types are identified by the letters "a" through "f". Type "a" is the line of symmetry where  $u_y=0$  (the specification of displacement boundary conditions is done in the usual way for finite element analyses). Boundary "b" is required to be far enough

removed that the soil has not yet felt the disturbance of the advancing plow, hence,  $u_x=0$  and  $u_y=0$ . For the plowing of a large soil mass, boundary "c" is required to be far enough removed that the soil does not detect the passing plow, thus,  $u_x=0$  and  $u_y=0$ . Boundary "d" is required to be far enough to the left that the soil has reached steady state behavior, thus

$$\begin{aligned}\frac{\partial u_x}{\partial t} &= 0 \\ \frac{\partial u_y}{\partial t} &= 0\end{aligned}\tag{24}$$

Using eq (2) this gives

$$\begin{aligned}\frac{\partial u_x}{\partial x} &= 0 \\ \frac{\partial u_y}{\partial x} &= 0\end{aligned}\tag{25}$$

The derivative boundary conditions, for a point such as "j" on boundary d, are specified using two point finite difference operators:

$$\begin{aligned}\frac{u_{xk}-u_{xj}}{x_k-x_j} &= 0 \\ \frac{u_{yk}-u_{uj}}{x_k-x_j} &= 0\end{aligned}\tag{26}$$

or

$$\begin{aligned}u_{xj}-u_{xk} &= 0 \\ u_{yj}-u_{yk} &= 0\end{aligned}\tag{27}$$

Because the soil model, the interface condition along the plow's surface and the cutting condition specification for the point at the tip of the plow, in general, all destroy the symmetry of the Jacobian matrix no attempt is made to preserve symmetry in the implementation of eq (27). The implementation of eq (27) is easily accomplished by replacing the finite element equilibrium equation 2j-1 by the equation that  $u_{xj}-u_{xk}=0$ . Similarly equation 2j is replaced by the equation  $u_{yj}-u_{yk}=0$ . The residuals just become the differences between the two displacements as estimated in the previous iteration; the Jacobian, the derivative of the residual, has entries of zeros and ones in the appropriate columns, i.e.,

$$\begin{aligned}
R_{2j-1} &= u_{xj}^{(I-1)} - u_{xk}^{(I-1)} \\
R_{2j} &= u_{yj}^{(I-1)} - u_{yk}^{(I-1)}
\end{aligned}
\tag{28}$$

and (all other elements are zero)

$$\begin{aligned}
J_{2j-1, 2j-1} &= 1 \\
J_{2j-1, 2k-1} &= -1 \\
J_{2j, 2j} &= 1 \\
J_{2j, 2k} &= -1
\end{aligned}
\tag{29}$$

Along the boundary "e" it is assumed that the trench produced by the plow remains open and, thus, the surface traction acting on the soil is just the water pressure if the plowing is below the surface of water and zero if it is in air; the resulting node point forces are added to the appropriate rows of the residual in the usual fashion.

Boundary segment f is the interface (excluding node  $p_c$ ) between the soil and the plow. Denote the equation of the surface of the plow by  $y=g(x^*)$  where  $x^*$  is measured from the plow tip, i.e,  $x^*=x-u_c$ , (recall that at the time of interest T, the location of the coordinate axis is specified so that the tip of the plow is at  $x=u_c$ , see Figures 2 , 3, 4 and 5). Along the soil-plow interface the frictional relationship between the effective, normal and tangential interface stresses must be enforced. This condition is approximated in terms of the "generalized forces" at the several nodes lying along the interface, such as node "i" in Figure 5. Both the virtual work and the minimum potential energy interpretation of the finite element equations can be used to give the interpretation of the negatives of the node point residuals ( $-R_{2i-1}$  and  $-R_{2i}$ ) as external reactions applied at the node (these of course should be zero at interior and stress free boundary nodes). Thus in Figure 5:

$$\begin{aligned}
F_{xi} &= -R_{2i-1\text{eff}} \\
F_{yi} &= -R_{2i\text{eff}}
\end{aligned}
\tag{30}$$

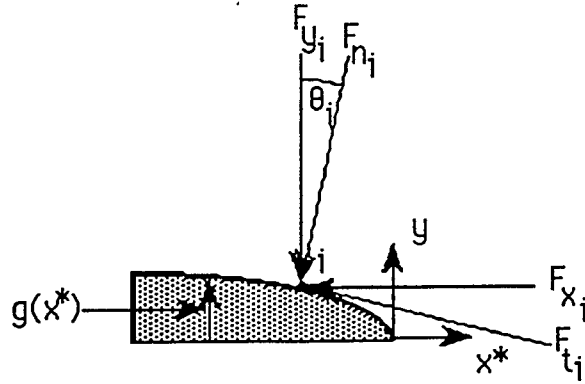


Figure 5. Components of generalized force at node  $i$ , due to action of soil on plow

The "eff" subscript indicates that the residuals must be those calculated using effective soil stresses (as opposed to using total stress as is the case for all other nodes where equilibrium equations are being generated). This mixed use of total and effective stress based residuals is accomplished by replacing those rows in the element residual and Jacobian matrices, which correspond to nodes along the interface, by rows calculated using the effective stresses (i.e., in eq 12 effective stresses are used and in eq 16 the effective tangent stiffness matrix is used, that is it is not augmented by the effective bulk modulus  $K_{eff}$ ). Equation 30 is now used to find the normal (n) and tangential components (t) of generalized force:

$$F_{n_i} = -R_{2i-1eff} \cos(\theta_i) - R_{2ieff} \sin(\theta_i) \quad (31)$$

$$F_{t_i} = R_{2i-1eff} \sin(\theta_i) - R_{2ieff} \cos(\theta_i)$$

Assuming that the interaction of the plow and the soil can be described by Coulomb friction with a coefficient of friction of "f" (the direction of  $F_{t_i}$  must be as shown in Figure 5 in order to oppose the motion of the plow):

$$F_{t_i} - f F_{n_i} = 0 \quad (32)$$

Introducing eq (31) into eq (32) gives:

$$R_{2i-1eff} [\sin(\theta_i) + f \cos(\theta_i)] - R_{2ieff} [\cos(\theta_i) - f \sin(\theta_i)] = 0 \quad (33)$$

This equation replaces the  $(2i-1)^{th}$  finite element equation. The new residual is the left hand side of eq (33) (where  $R_{2i-1eff}$  and  $R_{2ieff}$  are the old rows in the residual vector calculated using the effective soil stresses not the total) evaluated using the  $(I-1)$  estimate of the solution. Differentiating eq (33) with respect to the displacement components (and neglecting the dependence of  $\theta_i$  on  $u_c$ ) gives the new row in the global Jacobian matrix as a combination of the old rows where the R's of eq (33) are replaced by J's. The sin and cos of the angle  $\theta_i$  are calculated from geometry:

$$\begin{aligned}\sin(\theta_i) &= - \frac{g'}{\sqrt{1+(g')^2}} \\ \cos(\theta_i) &= \frac{1}{\sqrt{1+(g')^2}}\end{aligned}\tag{34}$$

Where  $g'$  is the derivative of  $g$  with respect to  $x^*$ . The values of  $g$  and  $g'$  are evaluated at the deformed location of node "i", i.e., at  $(x^*=x_i + u_{x_i} - u_c)$ , where the values of  $u_{x_i}$  and  $u_c$  are estimated using the results of the previous iteration. Because this dependency of  $\theta_i$  on the solution is ignored when forming the new row for the Jacobian it is an approximation to the true Jacobian (as noted previously such an approximation has no affect on the accuracy of the converged solution).

In addition to the frictional law, along the soil-plow interface, the conformity of the soil to the plow profile must be enforced (it is assumed that no separation occurs). Consider node "i" on the plow-soil interface, the compatibility condition between the plow and soil requires that:

$$u_{y_i} - g(x_i + u_{x_i} - u_c) = 0\tag{35}$$

This equation replaces the  $(2i)^{th}$  finite element equation. The residual is the left hand side of eq (35) (where the estimates from iteration  $I-1$  are used for  $u_{x_i}$  and  $u_c$ ). The new row for the Jacobian is found by differentiating the residual with respect to the displacement components (again the dependence on  $u_c$  is ignored in this process):

$$\begin{aligned}J_{2i,2i-1} &= -g' \\ J_{2i,2i} &= 1\end{aligned}\tag{36}$$

The above process assures that the deformed shape of the soil has the precise contour of the plow (assuming that the plow can be modeled as rigid). The precise fitting of the deformed soil to the plow profile may be judged to be a large deformation effect which could possibly be approximated by neglecting the difference between  $x$  and  $x^*$ . However, because the precise specification is quite a simple matter it is included in the analysis.

At node  $p_c$   $u_y=0$ . In addition, the "cutting criterion" for the soil must be specified. The determination of what the cutting state should be is a problem in fracture mechanics which to the Author's knowledge has not been addressed for soils. In this work two simple assumptions for the cutting state are discussed.

The first is that the soil being cut has reached a limiting value of effective tensile stress,  $\sigma_{fail}$ , across the surface being cut. This assumption is implemented by requiring that the effective normal (to the cut) stress  $\tau_{yyeff}$  should be equal to  $\sigma_{fail}$  at point  $p_c$ . This specification replaces the finite element equilibrium equation for the  $x$  direction at node  $p_c$  (the equilibrium equation contains the generalized force exerted on the soil by the tip of the plow, this force is not known a priori):

$$\tau_{yyeffp_c} - \sigma_{fail} = 0 \quad (37)$$

The quantity  $\tau_{yyeffp_c}$  is the effective normal stress at point  $p_c$  as predicted using the finite element approximation in element  $c$ . The prediction of the state of stress at point  $p_c$  in element  $c$  requires that the history dependency be evaluated using the strain states for all the corresponding points in the row of elements to the right of  $c$ . This process was previously discussed for the prediction of the states of stress at the element centers.

The  $(2p_c-1)$  residual is the left-hand side of eq (37) as predicted using the solution from the  $(I-1)$  iteration. The corresponding row in the Jacobian is found by differentiating the residual with respect to the displacement components (use is made of eq 15):

$$J_{2p_c-1,j} = D_{1,neff} B_{n,j} \quad (38)$$

Where  $D_{ijeff}$  are the tangent stiffness properties for the effective stress at the location of the third node (i.e.,  $p_c$ ) of element  $c$ . The components of the  $B$  matrix (eq 13) are found using eq (19) with  $k=3$ ; the repeated index  $n$  is summed from 1 to 3, and the subscript  $j$  in the Jacobian refers to the eight degrees of freedom for element  $c$  (i.e.,  $2m_i-1$  and  $2m_i$ , where  $m_i$  are the four nodes for element  $c$ ).

The second possibility for a cutting criterion at node  $p_c$  was suggested by Professor Kutter. It assumes that the soil is in a state of failure at point  $p_c$  as described by critical state soil mechanics. This condition assumes that cutting occurs when the effective stress state for the soil lies on the critical state line in  $I, J$  space ( $I$  is three times the mean effective stress and  $J$  is the second stress invariant for the deviatoric stress), i.e.,

$$J/I - M/\sqrt{27} = 0 \quad (39)$$

Where  $M$  is the slope of the critical state line in  $p, q$  space. The implementation of this cutting condition replaces the  $2p_{c-1}$  residual with the left-hand side of eq (39) evaluated at the third node of element  $c$ . The new row in the Jacobian is found by differentiating the residual (see the discussion of eq 39 for the meaning of  $J$ , etc.):

$$J_{2p_{c-1},j} = \left[ \frac{\sqrt{3}}{2J} (\beta_{1n} - I\beta_{2n} + 2\tau_{xx} D_{3n}) + M \beta_{2n} \right] B_{nj} \quad (40)$$

where

$$\beta_{1n} = \sum_{i=1}^2 \sigma_i D_{in_{eff}} \quad (41)$$

$$\beta_{2n} = \frac{1}{3} \sum_{i=1}^2 D_{in_{eff}}$$

With the boundary condition modifications of the finite element equations completed, the equations are ready for solution to give the Newton-Raphson correction to the node point displacements for iteration  $I$ , see eq (8). As noted previously the simultaneous equations are banded but non-symmetric. Iteration must be continued until convergence is achieved. Convergence is determined by the convergence of the plow force (described below).

It is of primary interest to determine the force required to propel the plow through the soil. This force is calculated as 2 times (to account for the fact that symmetry is used in the modeling of the plow) the sum of all the  $x$  components of the total generalized forces acting at the interface nodes along the soil-plow interface (including point  $p_c$ ) minus any force due to water pressure acting on the back of the plow.

## Example

The theory presented in the previous section has been implemented in a simple, non-production, finite element code intended for preliminary evaluation of the one-step, steady-state analysis concept. The analysis was implemented for both plane stress and plane strain conditions for a two-dimensional idealization of the plow problem. No attempt was made to accurately model the soil's inelasticity nor the pore-water pressure redistribution. Instead a linear viscoelastic model was used to model the soil, pore-water system (this choice was made in order to simplify and, thus, expedite this preliminary implementation). A linear viscoelastic model can qualitatively model the phenomena of soil inelasticity and pore-water pressure redistribution but can not do so quantitatively. Because it can qualitatively model these phenomena it is felt that its inclusion in the trial implementation will demonstrate whether or not the one-step, steady-state, inelastic analysis method is capable of handling these phenomena.

Because of the gross idealizations of two-dimensional geometry and linear viscoelastic behavior of the soil, pore-water system, no quantitative significance should be given to the results that are presented below. The example, however, will demonstrate the ability of the analysis method to handle either steady-state dynamic or quasi-static conditions; time-dependent, inelastic material behavior; interface friction between plow and soil; prescription of a failure condition for the soil at the tip of the plow; and the ability to accurately model arbitrarily shaped plows.

Two possible "cutting conditions" were suggested in the previous section. For the purpose of this example the imposed cutting condition was that the normal stress in the soil just ahead of the plow tip is zero (assumed zero tensile strength of the soil). While this condition may be a gross simplification of the actual situation, it is thought that comparisons of competing plow designs would still be meaningful and that quantitative predictions of plow force would still be relatively accurate (if the restrictions of two-dimensional geometry and idealized linear viscoelastic soil behavior were removed).

As an illustration of the potential capabilities of the one-step, steady-state analysis concept, the following problem was analyzed. A plane stress analysis was performed of a plow moving at constant velocity  $v_0$  through a layer of soil, see Figure 6. For the purpose of simplicity of data generation, the shape of the plow was taken to be a simple analytical curve of  $y_{blade} = t/2 [1 - e^{-\alpha(-x^*)}]$  where  $-L < x^* < 0$  (the coordinate  $x^*$  is measured from the tip of the plow, see discussion in the previous section). The length of the blade is denoted by  $L$ , the blade thickness is  $t$  and  $\alpha$  is a parameter that controls blade shape. As was stated it was for the sake of convenience that a simple equation was assumed for the shape of the blade; it is in fact a very simple matter to accommodate arbitrarily shaped plows.

The assumed linear viscoelastic properties of the soil, pore-water system are described by two relaxation functions, one in shear and the other in volume change, i.e.,

$$\Phi_G(t) = G_\infty + (G_0 - G_\infty) e^{-\alpha_G t}$$

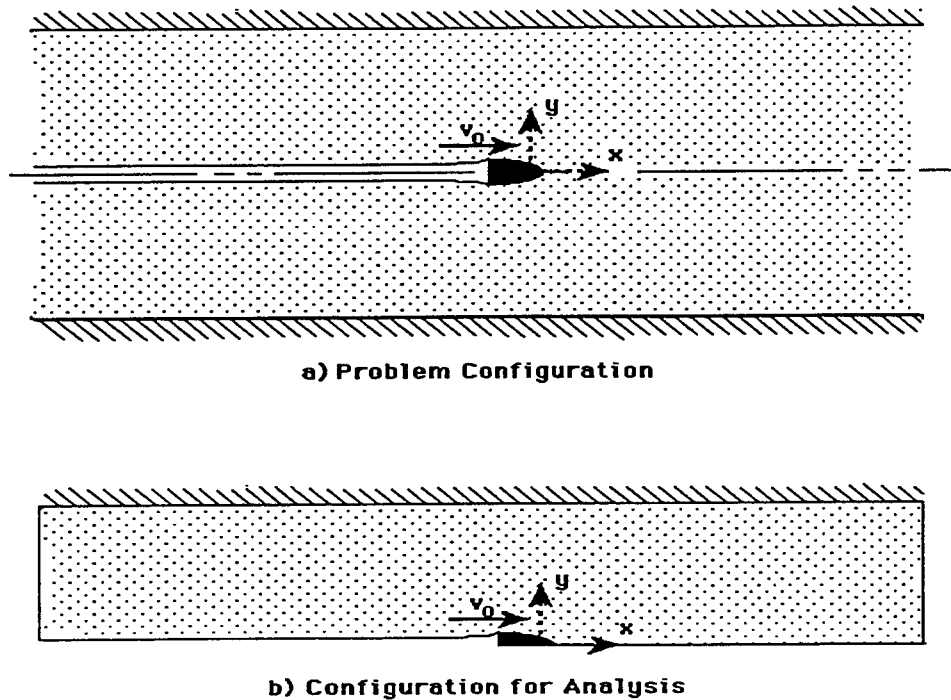
$$\Phi_B(t) = B_\infty + (B_0 - B_\infty) e^{-\alpha_B t}$$

The initial shear and bulk moduli are subscripted by "o", while the values at infinite time are subscripted by  $\infty$ . The rates of relaxation are controlled by  $\alpha_G$  and  $\alpha_B$ .

Using symmetry, the analysis can be restricted to half of the soil mass, see the lower part of Figure 6. The boundary condition at the outer edge of the soil mass (the tank wall for the Navy test, see [5,6]) was taken to be a fixed boundary.

Several analyses were run in order to determine the placement of the right boundary so that it was sufficiently far upstream of the plow not to feel any effect of the advancing plow. The boundary condition at this edge was taken to be fixed. Several analyses were run in order to determine the placement of the left boundary so that it was sufficiently far downstream of the plow to have reached a uniform state. The condition of uniformity was specified by setting the derivatives of the displacement components with respect to  $x$  to be zero at each of the node points along the left side boundary.

The boundary condition behind the plow and along the center-line of the soil is one of a stress free surface. The boundary condition ahead of the plow is the condition of symmetry. At the interface of the plow and the soil (except at the plow tip) the conditions are that the soil conforms to the shape of the plow and the generalized nodal shear forces equal the coefficient of friction  $f$  times the generalized nodal normal forces.



**Figure 6. Simple plow problem**

Finally at the very tip of the plow, the displacement  $u_y$  is required to be zero and the cutting condition is specified.

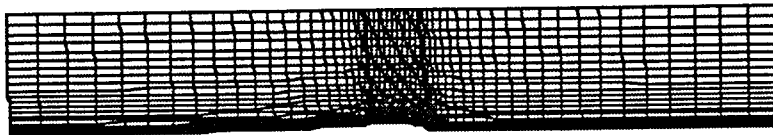
The total force  $F_0$ , required to move the plow at constant velocity through the soil, is calculated by summing the  $x$  components of the node point generalized forces for the nodes along the plow-soil interface (including the tip of the plow).

The coupling introduced into the simultaneous equations (for the steady-state, history dependent, finite element analysis) by the interchange of time and space when evaluating the history dependence of the soil properties was handled by moving the coupling terms to the right-hand side of the equations and approximating them by iteration. (Thus, the banded nature of the finite element equations was preserved.) Iteration was of course already necessary to model the nonlinearities of the problem (material inelasticity, the frictional interface condition and the enforcement of the cutting condition at the tip of the plow). As was previously noted, the equations are non-symmetric (this is also usually true for a time marching analysis if the correct Jacobian is used for the inelastic material).

Most of the parameters describing the example problem were selected with the Navy's laboratory test [5,6] in mind, however, because the assumed viscoelastic behavior for the soil can at best qualitatively represent the inelasticity of the soil and the movement of pore water, no effort was made to correlate this aspect of the problem description to the actual test conditions. The selected parameters were: viscoelastic soil properties of  $G_\infty=50$  psi,  $G_0=500$  psi,  $\alpha_G=30 \text{ sec}^{-1}$  and  $B_\infty=B_0=335,000$  psi; soil density of  $.081 \text{ lb/in}^3$ ; blade shape and dimensions of  $L=6.5$  in,  $t=3/8$  in and  $\alpha=0.7$ ; the dimensions of the soil mass were 45 inches to the left of the plow, 45 inches to the right of the plow and 15 inches for the half width.

A basic 4 element grid was selected and successively refined (maximum of approximately 2500 elements) until the predicted value of  $F_0$  converged (results from the 1300 element and 2500 element grids were nearly identical). From 5 to 10 iterations were required for convergence of the one-step, steady-state analysis. Plow velocities of 1, 3, 5 and 7 fps were considered; results for 5 fps are given in Figures 7-10.

Figure 7 shows the deformed mesh (displacements and plow thickness magnified by a factor of 5); all elements were rectangles in the undeformed mesh. Contour plots of the soil strains  $\gamma_{xy}$  and  $\epsilon_x$  are given in Figures 8 and 9, a contour plot of the soil stress  $\tau_{xy}$  is given in Figure 10. These plots clearly show that the length dimension has been taken large enough so as to reach undisturbed soil on the right and uniformity conditions on the left. Given in Figure 11 are plots of the plow force (per unit thickness of the plane stress body) as a function of plow velocity. The circles are for the plow configuration of  $\alpha=0.7$  and the case where acceleration of the soil is included; when the inertia is neglected the curve with square symbols was obtained. The curve with diamond symbols is for the case of a differently shaped plow ( $\alpha=1.2$ ); the two plow shapes are illustrated in Figure 12 (the vertical scale is exaggerated by about a factor of 7; the waviness of the lines are due to the plotting program).



mesh limits:

xmin -5.1500E+01

xmax 4.5000E+01

ymin 0.0000E+00

ymax 1.5000E+01

-----

status data:

nodes used 1440

elements 1357

Figure 7: Deformed mesh for example plow problem,  $v_0=5$  fps

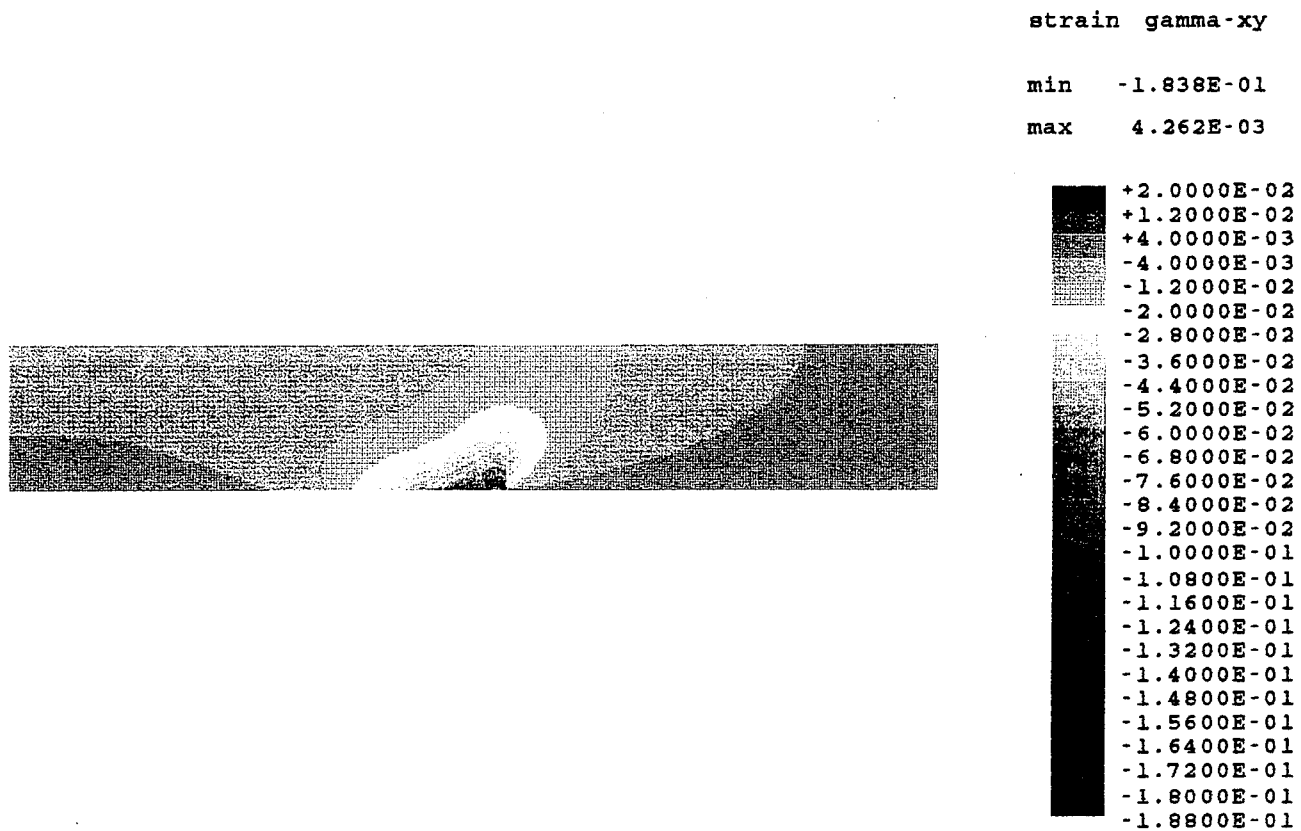


Figure 8: Contour plot of shear strain ( $\gamma_{xy}$ ) in soil,  $v_0=5$  fps

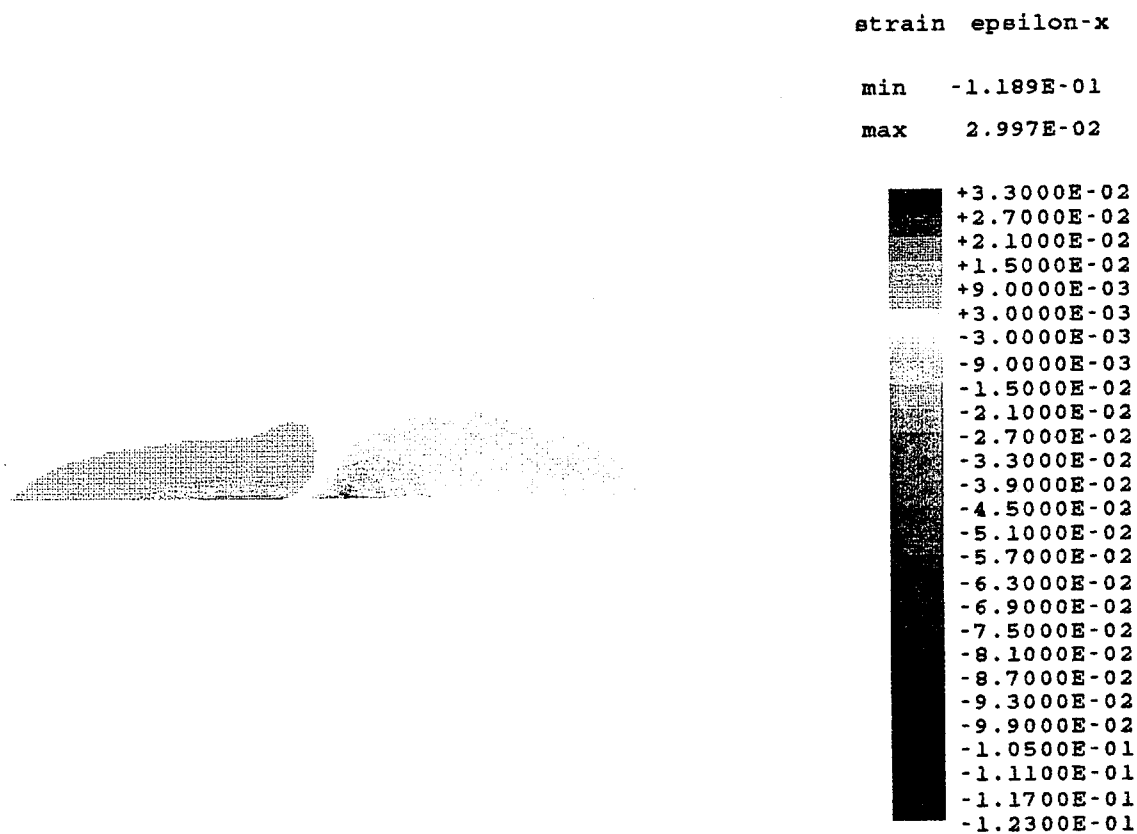


Figure 9: Contour plot of normal strain ( $\epsilon_x$ ) in soil,  $v_0=5$  fps

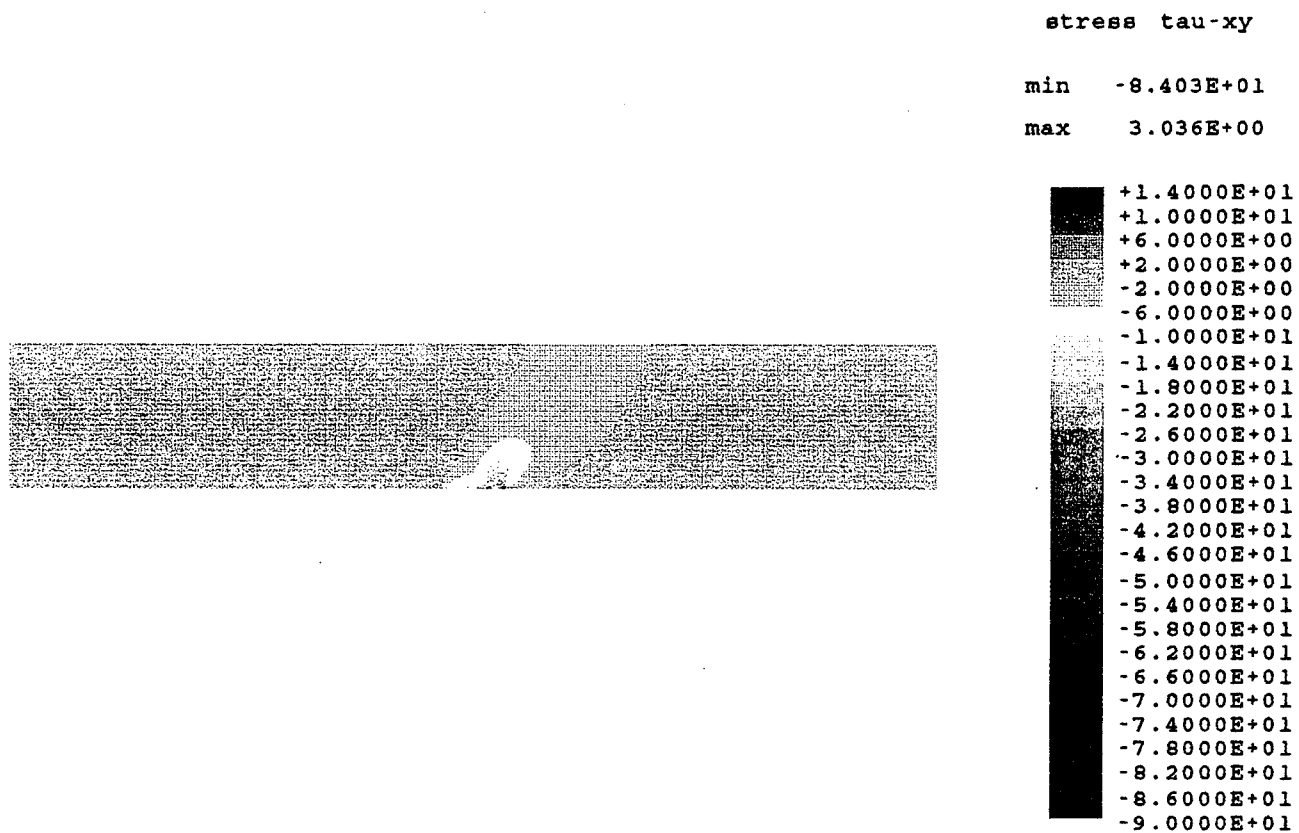


Figure 10: Contour plot of shear stress ( $\tau_{xy}$ ) in soil,  $v_0=5$  fps

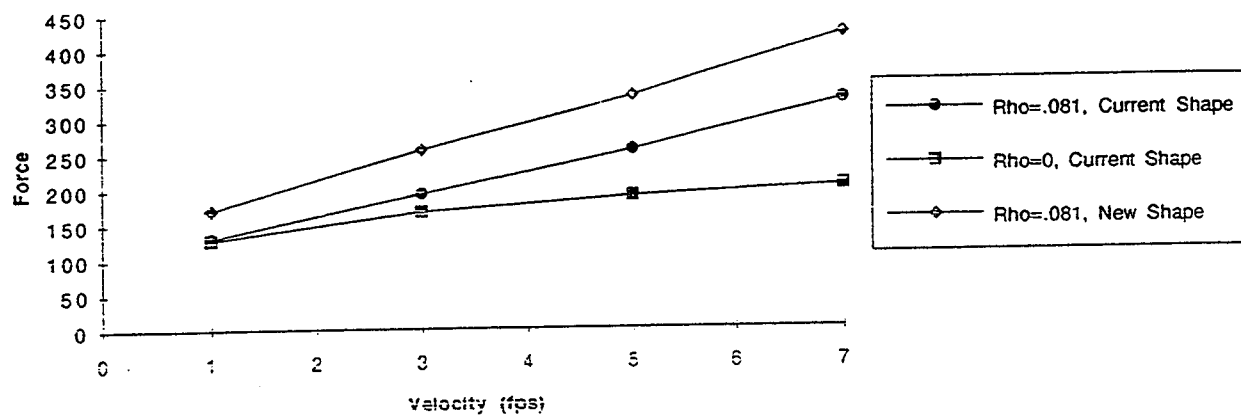


Figure 11: Dependence of plow force on plow velocity

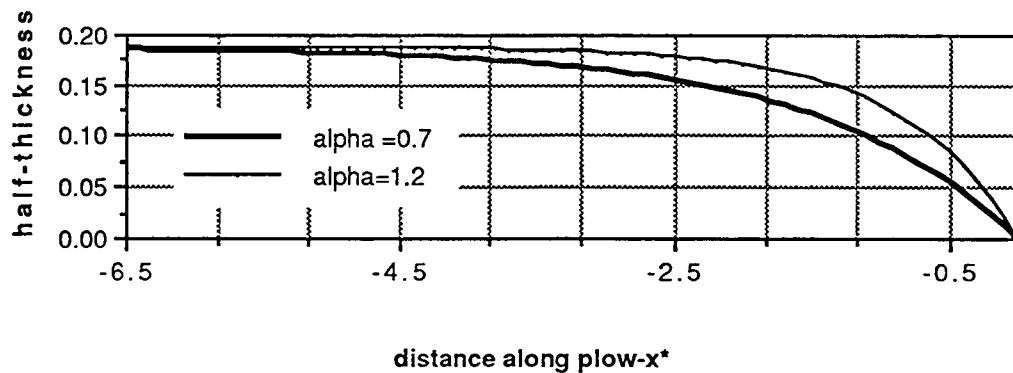


Figure 12. Shapes of plow for example problem

It is to be emphasized that no attempt was made to calibrate the viscoelastic material model in order to capture the actual soil inelasticity and pore-water pressure redistribution nor to pick its parameters so that  $F_0$  would agree with experimental observations (it appears that this would be possible but it would have very little significance).

What is important to observe from these results is the predicted dependence of the plow force  $F_0$  on the velocity,  $v_0$ , the rather dramatic dependence of the force upon the plow shape and the importance of including inertia effects. However, of even greater significance is the observation of the inexpensive nature of the analysis. As was mentioned only 5 to 10 iterations were required for convergence. Of course, because it is a steady-state analysis, no time stepping is required.

## Conclusions

From this preliminary study several conclusions can be drawn. Because of the very few iterations (and no time stepping) required, it should be entirely feasible to perform a three-dimensional, steady-state, inelasticity study of the plow problem including parameter studies for different plow shapes and plowing velocities. Thus, it appears that the one-step, steady-state analysis procedure offers a viable alternative to a multi-step, transient analysis of the

plow problem. However, because no three-dimensional code currently exists for the one-step, steady-state procedure, it would seem that the best way to proceed for the present study is to use an available commercial code to perform a multi-step, transient analysis.

Before any attempt is made to produce a production code for the one-step, steady-state procedure several items require further investigation. The failure (or cutting) condition in the soil at the tip of the plow (i.e. the state of the soil as it is cut by the plow tip to form a new surface) is not well understood and is an area that should receive further theoretical and experimental study (this same information is also required for a rigorous multi-step, transient analysis of the plow problem). The affect of using a realistic plasticity model for the soil on the rate of convergence of the iteration process must be studied. Finally, means for the incorporation of the flow of pore-water into the analysis must be developed.

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